Un'urna contiene 5 palline numerate

1, 2, 3, 4, 5 _ Si effettuano 2 estraz

1° caso	2° caso
con rimpiazzo	senza rimpiazzo

X = n° della 1ª pallina estratta

Y = n° ... 2ª ......

(X, Y)

ott 17-14:05

1° caso – con r.

$\boxed{\omega = (\omega_1, \omega_2)}$

$\omega_1 \in \{1, \dots, 5\}$

$\omega_2 \in \{1, \dots, 5\}$

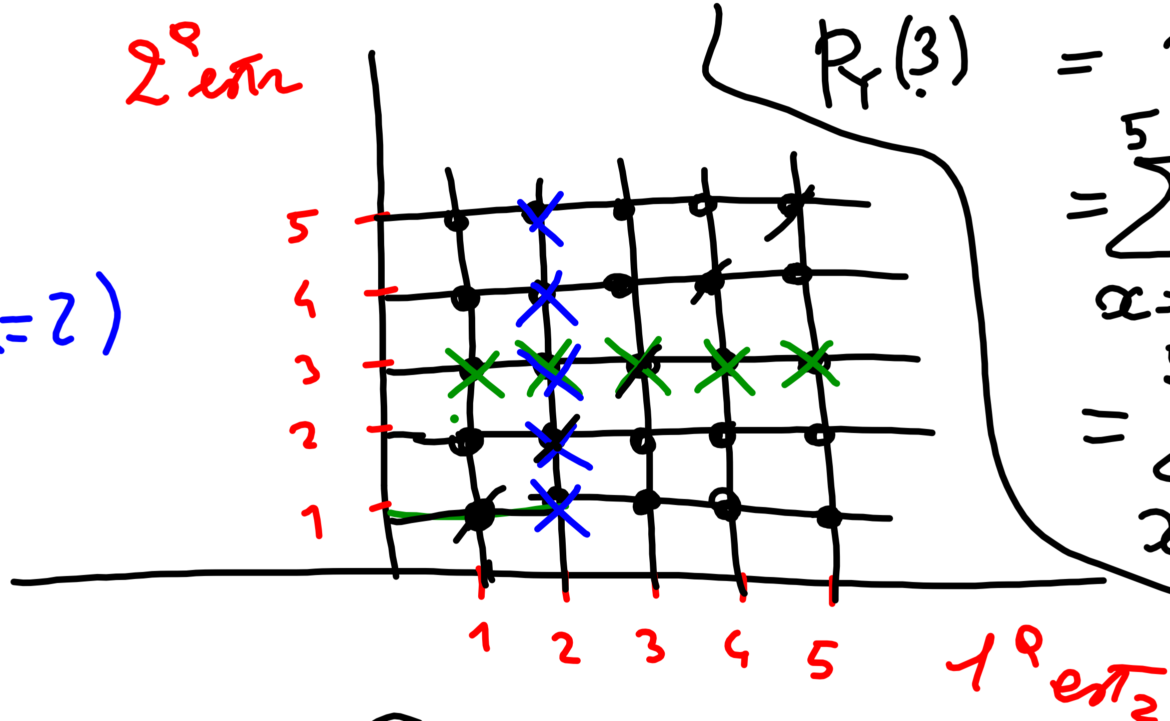
$(1,3) \quad (2,5)$

$(\Omega, \mathcal{A}, P)$

$\Omega = \{1,2,3,4,5\}^2, \ \mathcal{A} = \mathcal{P}(\Omega)$

$$P(\{\omega\}) = \frac{1}{\operatorname{card}\Omega} = \frac{1}{25}$$

ott 17-14:48



$P(X=2)$

$$p_Y(3) = P(Y=3) = \sum_{x=1}^{5} P(X=x, Y=3) = \sum_{x=1}^{5} p(x,3)$$

$$p_X(x) = P(X=x) = \begin{cases} \frac{1}{5} & x = 1,2,3,4,5 \\ 0 & \text{altrim.} \end{cases}$$

$P(X=1)$

$P(X=8)$

ott 17-14:52

2° caso. Senza Ω

$\Delta = \{\omega \in \{1,2,3,4,5\}^2 : \omega_1 = \omega_2\}$

$$\underline{\underline{\Omega = \{1,2,3,4,5\}^2 \setminus \Delta}}$$

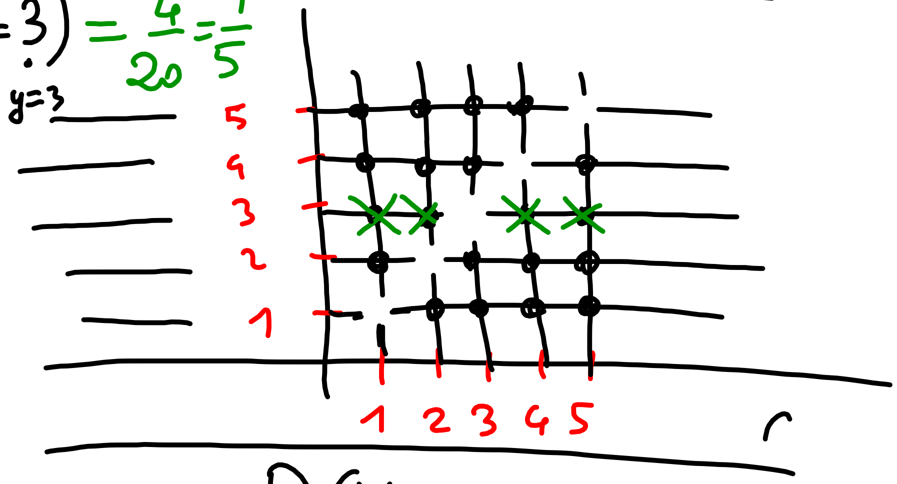
$(2,3)$

$(4,5)$

$$\mathcal{A} = \mathcal{P}(\Omega)$$

$$P(\{\omega\}) = \frac{1}{\operatorname{card}\Omega} = \frac{1}{20} = \frac{1}{5 \cdot 4}$$

$P(Y=3) = \frac{4}{20} = \frac{1}{5}$



$x=8$
$y=2$

$x=3$
$y=1$

$P(X=3, Y=1)$

$$p(x,y) = P(X=x, Y=y) = \begin{cases} \frac{1}{20} & (x,y) \in \{1,2,\dots,5\}^2 \setminus \Delta \\ 0 & \text{altrz.} \end{cases}$$

$$p_X(x) = \begin{cases} \frac{1}{5} & x \in \{1,\dots,5\} \\ 0 & \text{altr} \end{cases}$$

ott 17-15:16

$$p_Y(y) = \sum_{x \in \mathbb{R}} p(x,y) = \sum_{\substack{x=1 \\ \cancel{x \neq y}}}^{5} \frac{1}{20} = \frac{1}{5}$$

y

$$p_Y(y) = \begin{cases} \frac{1}{5} & y \in \{1,2,3,4,5\} \\ 0 & \text{—} \end{cases}$$

$$P(Y=3) = \frac{1}{5}$$

$$P(Y=3 \mid X=3) = 0$$

$$A = \{X=2\} \qquad B = \{Y=3\}$$

$$P(A \cap B) = P(A)P(B$$

$$\frac{1}{25} P(X=2, Y=3) = P(X=2) \cdot P(Y=3)$$

ott 17-15:13

Indipendenza di v. a.

X, Y 2 v. a.

$A = \{X \in \dot{I}\}$ $B = \{Y \in \dot{J}\}$

$\rightarrow P(X \in I, Y \in J) = P(X \in I)\, P(Y \in J)$

$\forall$ coppia I, J

Definizione. Le v. a. X e Y si dicono (tra loro) indipendenti

se $\boxed{P(X \in I, Y \in J) = P(X \in I)\, P(Y \in J)}$

$\forall\, I, J$ intervalli della retta $\mathbb{R}$

ott 17-15:22

Siano X, Y v. a. discrete con densità cong. $p(x,y)$ e marginali

$$p_X(x) \; ; \; p_Y(y)$$

$$p(x,y) = P(X=x, Y=y)$$

$$p_X(x) = P(X=x) \; ; \; p_Y(y) = P(Y=y)$$

<u>Proposizione</u>. X e Y sono indipend. se e solo se

prodotto tensoriale

$$\longrightarrow p(x,y) = p_X(x) \cdot p_Y(y) \quad \forall (x,y)$$

$f(x)$ $\quad g(x)$ $\quad h(x) = f(x) \cdot g(x)$

ott 17-15:29

Dim.

(1) Supp. che X e Y sono indip. Quindi

$$\to P(X \in I, Y \in J) = P(X \in I) \cdot P(Y \in J)$$

$\forall I, J$

$$\underbrace{I = \{x\}} \qquad \underbrace{J = \{y\}}$$

$$P(X \in \{x\}, Y \in \{y\}) = P(X \in \{x\}) P(Y \in \{y\})$$

$$\underbrace{P(X = x, Y = y)}_{p(x,y)} = \underbrace{P(X = x)}_{p_X(x)} \underbrace{P(Y = y)}_{p_Y(y)}$$

$$\to p(x,y) = p_X(x) \cdot p_Y(y)$$

ott 17-15:34

(2) Supp. che $\forall x, \forall y$

$$p(x,y) = p_X(x)\, p_Y(y)$$

Siano I e J due intervalli qualunque.

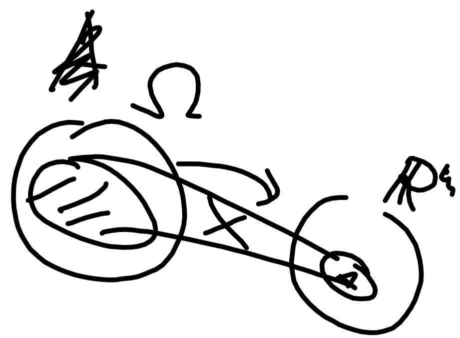
$$P(X \in I, Y \in J) = P\big((X,Y) \in \underbrace{I \times J}_{A}\big)$$

$$P(X \in A) = \sum_{x \in A} p(x) \longleftarrow A \subset \mathbb{R}^n$$

$$X = (X_1, \dots, X_n)$$

$x = (x_1 \dots x_n$

$$p(x) = p(x_1 \dots x_n)$$



Siano $X_1, X_2, \dots X_n$ n v. a.

<u>Def</u>. $X_1, \dots, X_n$ sono tra loro indipendenti se

$$P(X_1 \in I_1, X_2 \in I_2, \dots, X_n \in I_n) =$$
$$= P(X_1 \in I_1) \cdot \ldots \cdot P(X_n \in I_n)$$

$\forall I_1, I_2, \dots I_n$ intervalli della retta $\mathbb{R}$

$$P\big((X,Y)\in \underbrace{I\times J}\big) =$$

$$= \sum_{(x,y)\in I\times J} p(x,y)$$

$$P(X\in A) = \sum_{x\in A} p(x) \longleftarrow$$

$$P\big((X_1,\dots,X_m)\in A\big) = \sum_{(x_1,\dots x_m)\in A} p(x_1\dots x_m)$$

$m = 2$

$P(\setminus$

ott 17-15:43

Prop. X e Y sono indip. se e solo se

$$p(x,y) = p_X(x) \cdot p_Y(y) \quad \forall x,y$$

Prop. Siano $X_1, X_2, \dots, X_n$ v.a. discrete, con densità cong.

$p(x_1, \dots, x_n)$ e marginali

$$p_{X_1}(x_1), \; p_{X_2}(x_2) \; \dots \; p_{X_n}(x_n)$$

Allora $X_1, \dots, X_n$ sono indipend. se e solo se

$$p(x_1, \dots, x_n) = p_{X_1}(x_1) \cdot p_{X_2}(x_2) \cdot \ldots \cdot p_{X_n}(x_n)$$

ott 17-15:49

1° caso

$$p(x,y) = \begin{cases} \frac{1}{25} & (x,y) \in \underline{\{1 \dots 5\}^2} \\ 0 & \underline{\text{altr.}} \end{cases}$$

$$p_X(x) = \begin{cases} \frac{1}{5} & x \in \{1 \_ 5\} \\ 0 & \underline{\qquad} \end{cases}$$

$$p_Y(y) = \begin{cases} \frac{1}{5} & y \in \{1 \_ 5\} \\ 0 & \text{altr.} \end{cases}$$

ott 17-15:52

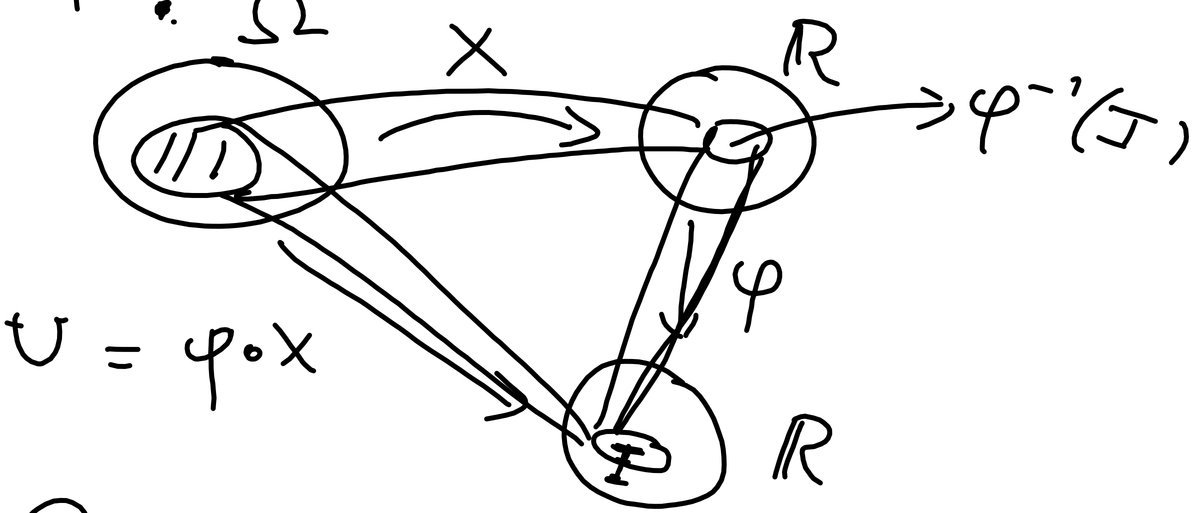
2° caso

$$p(x,y) = \begin{cases} \frac{1}{20} & (x,y) \in \{1,\dots,5\}^2 \setminus \Delta \\ 0 & \text{altr.} \end{cases}$$

$$p_X(x) = \begin{cases} \frac{1}{5} & x \in \{1..5\} \\ 0 \end{cases} \qquad p_Y(y) = \begin{cases} \frac{1}{5} & y \in \{1..5\} \\ 0 \end{cases}$$

X e Y indipend.

$U = \varphi \circ X$ $V = \psi \circ Y$



Proposizione Siano X e Y 2 v. a. indip e φ e ψ 2 funzioni $\overbrace{\mathbb{R}}^{\text{misurabili}} \to \mathbb{R}$

Allora $U = \varphi \circ X$ e $V = \psi \circ Y$ sono 2 v. a. indipendenti

Dim.

$$\boxed{P(U \in I, V \in J) =}$$

$$= P(\varphi \circ X \in I, \psi \circ Y \in J) =$$

$$= P(X \in \underbrace{\varphi^{-1}(I)}, Y \in \underbrace{\psi^{-1}(J)}) =$$

$$= P(X \in \varphi^{-1}(I))\, P(Y \in \psi^{-1}(J))$$

$$= P(\varphi \circ X \in I)\, P(\psi \circ Y \in J)$$

$$\forall I, J = \boxed{P(U \in I) \cdot P(V \in J)}$$

ott 17-16:01

$$\varphi(X_1, \ldots, X_m) = U$$

$$\psi(Y_1, \ldots, Y_n) = V$$

Se $\{X_1, \ldots, X_m, Y_1 \ldots Y_n\}$ sono

v.a. indipendenti, allora

U e V sono indipendenti

ott 17-16:05

$$P(X\in I, Y\in J) = P((X,Y)\in I\times J) =$$

$$= \sum_{(x,y)\in I\times J} p(x,y) \overset{\text{ipotesi}}{=} \sum_{(x,y)\in I\times J} p_X(x)\cdot p_Y(y)$$

$$= \left(\sum_{x\in I} p_X(x)\right)\left(\sum_{y\in J} p_Y(y)\right) =$$

$$= P(X\in I)\cdot P(Y\in J)$$

ott 17-15:02

$$p_Y(y) = P(Y=y) = \begin{cases} \frac{1}{5} & y = 1,2,3,4,5 \\ 0 & \text{alh.} \end{cases}$$

$P(Y=8)=0$

$P(Y=1)=\frac{1}{5}$

$P(X=1, Y=2)$

$P(X=8, Y=7)$

$P(X=8, Y=2)$

$(8,2)$

$$p(x,y) = P(X=x, Y=y) = \begin{cases} \frac{1}{25} & \longrightarrow (x,y) \in \{1,2,3,4,5\}^2 \\ 0 & \text{alh.} \end{cases}$$

ott 17-14:55